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APPLICATION NOTE 1140

Receiver Sensitivity Equation for Spread Spectrum Systems

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Abstract: The following application note provides insight into how the sensitivity of a spread spectrum application is defined and how the desired sensitivity level for digital communication receivers can be calculated. This technical paper provides a step-by-step development of the receiver sensitivity equation and concludes with numeric example, putting its mathematical definition to the test.

In a spread spectrum digital communication receiver, the relationship between the link metric E_b/N_o (bit energy to noise power spectral density ratio) and the RF power level to achieve the desired receiver sensitivity is derived from the standard noise factor definition, F. Resulting in the receiver sensitivity equation, this relationship is used by RF designers for CDMA, WCDMA cellular receivers and other spread spectrum systems. It allows the designer to determine the receiver parameter trade-offs in a spread spectrum link budget for any given input signal level.

Click here for an overview of the wireless components used in a typical radio transceiver.

Deriving E_b/N_o Relationship From Noise Factor, F

By definition, F is the ratio of signal-to-noise at the input of a device (a single stage, multiple stages, or the complete receiver) to the signal-to-noise at the output of the same device (**Figure 1**). Since noise varies in an unpredictable manner from one point in time to the next, taking the ratio of the mean-square signal to the mean-square noise forms the signal-to-noise ratio (SNR).

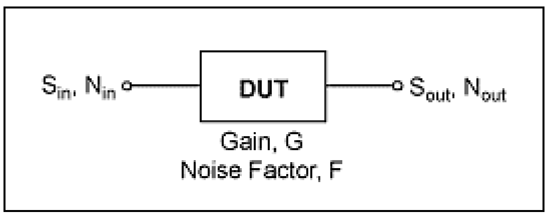

Figure 1.

Following are the definitions for parameters used in Figure 1 and for the sensitivity equation:

S_{in} = available input signal power (W)
N_{in} = available input thermal noise power (W) = KTB_{RF} where:
K = Boltzmann's constant = 1.381×10^{-23} W/Hz/K,
T = 290K at room temperature and
B_{RF} = RF carrier bandwidth (Hz) = chip rate for the spread-spectrum system
S_{out} = available output signal power (W)
N_{out} = available output noise power (W)
G = device gain (numeric)
F = device noise factor (numeric)

F is defined as follows

$$F = (S_{in} / N_{in}) / (S_{out} / N_{out}) = (S_{in} / N_{in}) \times (N_{out} / S_{out})$$

Solving for N_{out} in terms of the input noise, N_{in}

$$N_{out} = (F \times N_{in} \times S_{out}) / S_{in} \text{ where } S_{out} = G \times S_{in}$$

results in

$$N_{out} = F \times N_{in} \times G$$

The average modulating signal power is defined as $S = E_b / T$, where E_b is the energy in the bit interval in W-s and T is the bit time interval in seconds.

The relationship for the average modulating signal power for the user's data rate is calculated as follows

$$1 / T = \text{user bit rate, } R_{bit} \text{ in Hz, which results in } S_{in} = E_b \times R_{bit}$$

Based on the previous equations, the signal-to-noise ratio at the output of the device in terms of E_b/N_o is

$$S_{out} / N_{out} = (S_{in} \times G) / (N_{in} \times G \times F) =$$
$$S_{in} / (N_{in} \times F) =$$
$$(E_b \times R_{bit}) / (KTB_{RF} \times F) =$$
$$(E_b / KTF) \times (R_{bit} / B_{RF}),$$

where KTF represents the noise power (N_o) in a 1-bit interval.

Therefore,

$$S_{out} / N_{out} = E_b/N_o \times R_{bit} / B_{RF}$$

With the RF bandwidth, B_{RF} being equal to the chip rate W in a spread spectrum system, the processing gain ($PG = W/R_{bit}$) can be defined as

$$PG = B_{RF} / R_{bit}$$

Therefore, $R_{bit} / B_{RF} = 1/PG$, which results in the output signal-to-noise ratio

$$S_{out} / N_{out} = E_b/N_o \times 1 / PG.$$

Note: For a system that is not spread in bandwidth (that is W = R), the value of E /N is numerically bit b o

equal to SNR.

Receiver Sensitivity Equation

To determine SNR for a given input signal level, solve for S_{in} from the noise factor equation

$$F = (S_{in} / N_{in}) / (S_{out} / N_{out}) \text{ or } F = (S_{in} / N_{in}) \times (N_{out} / S_{out})$$
$$S_{in} = F \times N_{in} \times (S_{out} / N_{out})$$

S_{in} can also be expressed as

$$S_{in} = F \times KTB_{RF} \times E_b/N_o \times 1/PG$$

In a more useful logarithmic form, take 10 × log of each term yielding units of dB or dBm. With the noise figure NF (dB) = 10 × log (F), this leads to the following receiver sensitivity equation

$$Sin\ (dBm) = NF\ (dB) + KTB_{RF}\ (dBm) + E_b/N_o\ (dB) - PG\ (dB)$$

Numeric Example

The following example is for a spread-spectrum WCDMA cellular base station receiver. Though the receiver sensitivity equation holds true for all levels of input signal level, this example uses the maximum specified input signal power at the minimum specified sensitivity in percent of the Bit Error Rate (%BER) for a given E_b/N_o. Following are the conditions for this numeric example:

- The maximum specified input signal level has to meet the minimum specified system sensitivity for a 12.2kbps digital voice data rate signal at -121dBm.
- The specified BER (0.1%) can be achieved for an E_b/N_o value of 5dB for the QPSK modulated signal.
- The RF bandwidth is equal to the chip rate, which is 3.84MHz.
- $KTB_{RF}(log) = 10 \times log(1.381 \times 10^{-23}$ W/Hz/K × 290K × 3.84MHz × 1000mW/W) = -108.13dBm.
- For a specified user data rate of R_{bit} equal to 12.2kbps, PG is $PG = R_{chip} / R_{bit} = 314.75_{numeric}$ or $25dB_{log}$.
- Substituting these values and solving for $S_{out} / N_{out} = E_b/N_o \times R_{bit} / B_{RF}$ yields the output signal-to-noise ratio as 5dB - 25dB = -20dB. This shows that spread spectrum systems actually operate with negative SNR for a spread bandwidth.

To find the maximum allowable receiver noise figure, which meets the minimum specified sensitivity simply solve for NF_{max}, using the receiver sensitivity equation.

$$S_{in}\ (dBm) = NF\ (dB) + KTB_{RF}\ (dBm) + E_b/N_o\ (dB) - PG\ (dB)$$

The following steps and **Figure 2** provide additional guidance to finding NF_{max}:

Step 1: The maximum specified RF input signal at desired sensitivity is -121dBm for WCDMA.

Step 2: Subtract the E_b/N_o value of 5dB, which yields the maximum allowable noise level in the user bandwidth (12.2kHz) of -126dBm.

Step 3: Determine the maximum noise level in the RF carrier bandwidth by adding a processing gain of 25dB, which results in the maximum allowable noise level of -101dBm.

Step 4: Subtract the maximum allowable noise level from device input noise level resulting in NF_{max} = 7.1dB.

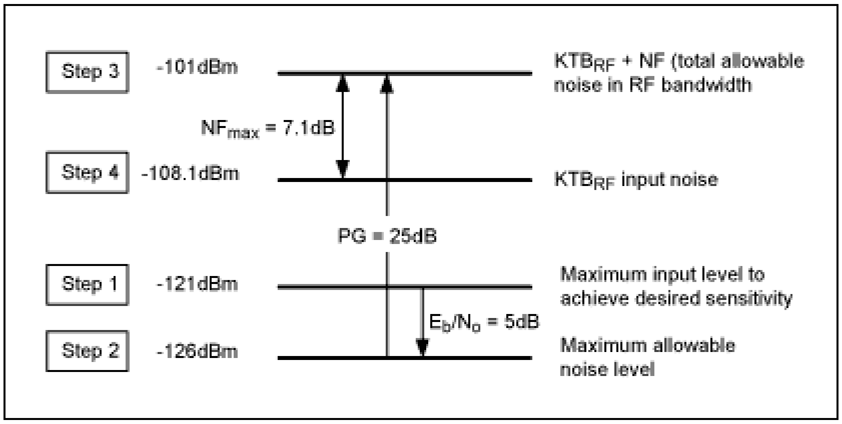

Figure 2.

Note: If a more efficient detector is used in the receiver design that only requires an E_b/N_o value of 3dB instead of 5dB, a receiver sensitivity level of -123dBm can be obtained for the same receiver NF_{max} of 7.1dB. On the other hand, a higher NF_{max} of 9.1dB can still be tolerated and meet the maximum specified input signal level of -121dBm at sensitivity for the reduced E_b/N_o value.

Conclusion

Using the receiver sensitivity equation,

$$S_{in} \text{ (dBm)} = NF \text{ (dB)} + KTB_{RF} \text{ (dBm)} + E_b/N_o \text{ (dB)} - PG \text{ (dB)}$$

derived from the noise factor definition, designers can determine the receiver parameter trade-offs in a spread spectrum link budget for any given input signal level, which makes it particularly useful to determine system sensitivity.

References

1. *CDMA Systems Engineering Handbook*, Jhong Sam Lee & Leonard E. Miller, Artech House Publishers, 1998.
2. *CDMA RF System Engineering*, Samuel C. Yang, Artech House Publishers, 1998.